

Interparticle potential energy for D -dimensional electromagnetic models from the corresponding scalar ones

Antonio Accioly,^{*} José Helayël-Neto,[†] Gilson Correia,[‡] Gustavo Brito,[§] José de Almeida,[¶] and Wallace Herdy^{**}
*Laboratório de Física Experimental (LAFEX), Centro Brasileiro de Pesquisas Físicas (CBPF),
 Rua Dr. Xavier Sigaud 150, Urca, 22290-180, Rio de Janeiro, RJ, Brazil*

(Dated: May 18, 2016)

Using a method based on the generating functional plus a kind of “correspondence principle” — which acts as a bridge between the electromagnetic and scalar fields — it is shown that the interparticle potential energy concerning a given D -dimensional electromagnetic model can be obtained in a simple way from that related to the corresponding scalar system. The D -dimensional electromagnetic potential for a general model containing higher derivatives is then found from the corresponding scalar one and the behavior of the former is analyzed at large as well as small distances. In addition, we investigate the presence of ghosts in the four-dimensional version of the potential associated with the model above and analyze the reason why the Coulomb singularity is absent from this system. The no-go theorem by Ostrogradski is demystified as well.

I. INTRODUCTION

Time and again new interesting and important electromagnetic models are proposed aiming to overcome the hurdles that are inherent in the theoretical description of the electromagnetic interactions. Nonetheless, all these systems have inevitably to reproduce in the nonrelativistic limit the Coulombian potential energy plus a possible correction. Now, bearing in mind that this potential energy is singular at the origin, it is easy to understand the great importance of searching for electromagnetic models in which this singularity is absent. Accordingly, it is extremely important to have available an easy prescription on hand for finding the potential energy for those new electromagnetic models so that their behavior at small distances can be analyzed promptly and efficiently.

In this vein, a simple prescription for computing the mentioned potential was recently built up [1]. Our primary aim here is to show that if we start from the scalar theory corresponding to the electromagnetic model we want to analyze and which is obtained by utilizing a kind of “correspondence principle” that acts as a bridge between the electromagnetic and scalar fields, we will arrive, *mutatis mutandis*, at the potential for the electromagnetic system described in [1]. Of course, to work with scalar theories is always much easier than with electromagnetic ones, which is a good argument in favor of our method.

On the other hand, electromagnetic theories lacking the Coulomb singularity can often be obtained by adding higher-order terms to the Maxwell Lagrangian. Why is this so? Because the higher-derivative terms are responsible for giving origin to a potential with a sign that is op-

posite to the Coulomb one and, as a result, at the origin this correction to the Coulomb potential is responsible in general for canceling out the contribution coming from the aforementioned potential. Now, since the higher-derivative potential contributes with an energy that has a sign which is the opposite of that concerning the photon, we are in the presence of a ghost. Note, however, that renormalizable higher-order theories can be seen in general as very satisfactory effective field theories below the Planck scale [2–4]. Our second aim in this paper is to discuss the issue of ghosts in higher-derivative theories. In particular, we shall discuss why the Ostrogradski’s no-go theorem [5] cannot be used to discard gauge theories.

The article is organized as follows.

In the next section we discuss the method for computing the D -dimensional electromagnetic interparticle potential energy from the corresponding scalar one.

In Sec. III, we compute the D -dimensional interparticle potential energy for a general electromagnetic model containing higher derivatives utilizing the related scalar system and analyze the behavior of the former both at large and small distances.

We investigate in Sec. IV the issue of ghosts in higher-derivative models and demystify the no-go theorem by Ostrogradski. In particular, we discuss the presence of ghosts in the four-dimensional version of the potential found in Sec. III and analyze the reason why the Coulomb singularity is absent from the aforementioned electromagnetic systems.

Our conclusions are presented in Sec. V.

Technical details are relegated to the Appendix.

We use natural units throughout and our Minkowski metric is $\text{diag}(1, -1, \dots, -1)$.

II. THE METHOD

As is well known, the generating functional for the connected Feynman diagrams $W_D(J)$ is related to the generating functional $Z_D(J)$ for a scalar theory, by $Z_D(J) = e^{iW_D(J)}$ [6–8], where

^{*} accioly@cbpf.br
[†] helayel@cbpf.br
[‡] gilson@cbpf.br
[§] gpbrito@cbpf.br
[¶] josejr@cbpf.br
^{**} wallacew@cbpf.br

$$W_D(J) = -\frac{1}{2} \iint d^D x d^D y J(x) D(x-y) J(y). \quad (1)$$

Here $J(x)$ and $D(x-y)$ are, respectively, the external current and the propagator.

Now, keeping in mind that

$$\begin{aligned} D(x-y) &= \int \frac{d^D k}{(2\pi)^D} e^{ik(x-y)} D(k), \\ J(k) &= \int d^D x e^{-ikx} J(x), \end{aligned}$$

we promptly obtain

$$W_D(J) = -\frac{1}{2} \int \frac{d^D k}{(2\pi)^D} J(k)^* D(k) J(k). \quad (2)$$

Assuming then that the external current is time independent, we get from (2)

$$\begin{aligned} W_D(J) &= -\frac{1}{2} \int \frac{d^D k}{(2\pi)^{D-1}} \left[\delta(k^0) T D(k) \int \int d^{D-1} \mathbf{x} \right. \\ &\quad \left. \times d^{D-1} \mathbf{y} e^{i\mathbf{k} \cdot (\mathbf{y} - \mathbf{x})} J(\mathbf{x}) J(\mathbf{y}) \right], \end{aligned} \quad (3)$$

where the time interval T is produced by the factor $\int dx^0$.

Simple algebraic manipulations, on the other hand, reduces (3) to the form

$$W_D(J) = -T \int \frac{d^{D-1} \mathbf{k}}{(2\pi)^{D-1}} D(\mathbf{k}) \Delta(\mathbf{k}), \quad (4)$$

where $D(\mathbf{k}) \equiv D(k)|_{k^0=0}$, and

$$\Delta(\mathbf{k}) \equiv \int \int d^{D-1} \mathbf{x} d^{D-1} \mathbf{y} e^{i\mathbf{k} \cdot (\mathbf{y} - \mathbf{x})} \frac{J(\mathbf{x}) J(\mathbf{y})}{2}. \quad (5)$$

In the specific case of two scalar charges σ_1 and σ_2 located, respectively, at \mathbf{a}_1 and \mathbf{a}_2 , the current assumes the form

$$J(\mathbf{x}) = \sigma_1 \delta^{D-1}(\mathbf{x} - \mathbf{a}_1) + \sigma_2 \delta^{D-1}(\mathbf{x} - \mathbf{a}_2). \quad (6)$$

Therefore,

$$\Delta(\mathbf{k}) = \sigma_1 \sigma_2 e^{i\mathbf{k} \cdot \mathbf{r}}, \quad (7)$$

where $\mathbf{r} = \mathbf{a}_2 - \mathbf{a}_1$, and

$$W_D(J) = -T \frac{\sigma_1 \sigma_2}{(2\pi)^{D-1}} \int d^{D-1} \mathbf{k} e^{i\mathbf{k} \cdot \mathbf{r}} D(\mathbf{k}). \quad (8)$$

Bearing in mind that

$$Z_D(J) = \langle 0 | e^{-iH_D T} | 0 \rangle = e^{-iE_D^{(\text{scal})} T}, \quad (9)$$

which implies that

$$E_D^{(\text{scal})} = -\frac{W_D(J)}{T}, \quad (10)$$

we come to the conclusion that the D -dimensional potential energy can be computed through the simple expression

$$E_D^{(\text{scal})}(r) = \frac{\sigma_1 \sigma_2}{(2\pi)^{D-1}} \int d^{D-1} \mathbf{k} e^{i\mathbf{k} \cdot \mathbf{r}} D(\mathbf{k}). \quad (11)$$

It is worth noting that Zee [6] utilized Eq. (1) to show that in $D = 4$ two static real scalar sources attract each other by virtue of their coupling to the scalar field.

On the other hand, the interparticle potential energy for the interaction of two point-like static electric charges in the framework of a D -dimensional electromagnetic model is given by [1]

$$E_D^{(\text{electr})}(r) = \frac{Q_1 Q_2}{(2\pi)^{D-1}} \int d^{D-1} \mathbf{k} e^{i\mathbf{k} \cdot \mathbf{r}} \mathcal{P}_{00}(\mathbf{k}), \quad (12)$$

where $\mathcal{P}_{\mu\nu}(k)$ is the “propagator” in momentum space obtained by neglecting all terms of the usual Feynman propagator in momentum space that are orthogonal to the external conserved currents and $\mathcal{P}_{\mu\nu}(\mathbf{k}) \equiv \mathcal{P}_{\mu\nu}(k)|_{k^0=0}$. We remark that in the deduction of (12) it was assumed that the external current is conserved.

Let us then show that $\mathcal{P}_{00}(\mathbf{k}) = D(\mathbf{k})$. To do that we recall that the Lagrangian associated with a D -dimensional electromagnetic theory can be written as

$$\mathcal{L}^{(\text{electr})}(x) = \frac{1}{2} A^\mu(x) O_{\mu\nu}(x) A^\nu(x). \quad (13)$$

Here, $O_{\mu\nu}(x) \equiv a(x)\theta_{\mu\nu}(x) + b(x)\omega_{\mu\nu}(x)$ is the wave operator and $\theta_{\mu\nu} \equiv \eta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\square}$ and $\omega_{\mu\nu} \equiv \frac{\partial_\mu \partial_\nu}{\square}$ are the usual vectorial projector operators. Accordingly, the corresponding propagator is given by

$$O_{\mu\nu}^{-1}(x) = \frac{1}{a} \theta_{\mu\nu} + \frac{1}{b} \omega_{\mu\nu}, \quad (14)$$

which in momentum space can be written as

$$O_{\mu\nu}^{-1}(k) = \frac{1}{a(k)} \theta_{\mu\nu}(k) + \frac{1}{b(k)} \omega_{\mu\nu}(k), \quad (15)$$

where $\theta_{\mu\nu}(k) \equiv \eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}$ and $\omega_{\mu\nu}(k) \equiv \frac{k_\mu k_\nu}{k^2}$.

Thus, $\mathcal{P}_{\mu\nu}(\mathbf{k}) = \frac{1}{a(\mathbf{k})} \eta_{\mu\nu}$ and, as a consequence,

$$\mathcal{P}_{00}(\mathbf{k}) = \frac{1}{a(\mathbf{k})}. \quad (16)$$

We formulate in the following a kind of correspondence principle in order to connect the electromagnetic and scalar fields. Technically, this link can be achieved via the correspondence below

$$A^\mu (\text{or } A_\mu) \longrightarrow \phi,$$

$$\partial_\nu A^\mu \partial_\mu A^\nu \longrightarrow 0,$$

$$J^\mu (\text{or } J_\mu) \longrightarrow J.$$

We call attention to the fact that in the second expression listed above, $\partial_\nu A^\mu \partial_\mu A^\nu$ stands for all the expressions that can be obtained from it via integration by parts. We also remark that as a straightforward consequence of the mentioned correspondence principle $-\frac{1}{4}F_{\mu\nu}^2 \longrightarrow -\frac{1}{2}\partial_\mu \phi \partial^\mu \phi$, where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength.

Now, if we take the above mentioned correspondence principle into account, we promptly obtain from (13)

$$\mathcal{L}^{(\text{scal})}(x) = \frac{1}{2}\phi(x)a(x)\phi(x). \quad (17)$$

Therefore, the scalar propagator in momentum space reads

$$D(k) = \frac{1}{a(k)}. \quad (18)$$

As a result, $D(\mathbf{k}) = \frac{1}{a(\mathbf{k})}$, implying that $\mathcal{P}_{00}(\mathbf{k}) = D(\mathbf{k})$.

Accordingly, the steps to be followed to arrive at the D -dimensional interparticle potential energy for an electromagnetic model from the related scalar system, are:

- A. Use the correspondence principle to find the scalar Lagrangian from the original electromagnetic one.
- B. Compute $D(\mathbf{k})$.
- C. Calculate $\int d^{D-1}\mathbf{k} e^{i\mathbf{k}\cdot\mathbf{r}} D(\mathbf{k})$.
- D. Obtain the D -dimensional interparticle potential energy ($E_D^{(\text{electr})}(r)$) for the interaction of two static electric charges (Q_1 and Q_2) via the expression

$$\frac{Q_1 Q_2}{(2\pi)^{D-1}} \int d^{D-1}\mathbf{k} e^{i\mathbf{k}\cdot\mathbf{r}} D(\mathbf{k}).$$

A comparison between the scalar Lagrangian obtained via the correspondence principle and that related to the standard scalar field, clearly shows they differ by an overall minus sign (the correspondence principle changes the sign of the usual scalar Lagrangian); as a consequence, the standard scalar potential energy between two static real scalar charges (σ_1 and σ_2) must be computed through the expression

$$E_D^{(\text{standscal})}(r) = -\frac{\sigma_1 \sigma_2}{(2\pi)^{D-1}} \int d^{D-1}\mathbf{k} e^{i\mathbf{k}\cdot\mathbf{r}} D(\mathbf{k}). \quad (19)$$

III. D -DIMENSIONAL ELECTROMAGNETIC POTENTIAL ENERGY FOR A GENERAL MODEL CONTAINING HIGHER DERIVATIVES FROM THE CORRESPONDING SCALAR ONE

To test the efficacy and simplicity of the method developed in the last section we shall find, using the aforementioned prescription, the D -dimensional potential energy for the general electromagnetic model defined by the Lagrangian

$$\mathcal{L}^{(\text{electr})} = -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}M^2 A_\mu^2 - \frac{1}{4m^2}F_{\mu\nu}\square F^{\mu\nu} - J^\mu A_\mu, \quad (20)$$

where M and m are parameters with mass dimension. Note that if $M \rightarrow 0$ we recover Lee-Wick electrodynamics which has been object of increasing research as time goes by [9–33]. Actually, the electromagnetic model we are considering is nothing but a natural extension of the Lee-Wick system. Since the method we have developed assumes that the external current is conserved, for completeness sake, we shall find beforehand the constraint on the field A^μ owed to the mentioned conservation law.

From (20), we immediately obtain

$$\left[1 + \frac{\square}{m^2}\right] \partial_\mu F^{\mu\nu} + M^2 A^\nu = J^\nu. \quad (21)$$

On the other hand, requiring conservation of the external current we arrive at the conclusion that $\partial_\mu A^\mu = 0$. Note that if we assume that the external current concerning Proca electrodynamics is conserved, we obtain a constraint on A^μ that coincides with the one we have just found.

Taking the aforementioned restriction into account, we get from (21) a generalized wave equation for the field A^μ , i.e.,

$$\left(\square + \frac{\square\square}{m^2} + M^2\right) A^\mu = 0. \quad (22)$$

After this little digression, let us return to our main theme: the computation of the potential energy for an

electromagnetic model from the corresponding scalar system.

The scalar Lagrangian corresponding to (20) can be easily obtained by means of the correspondence principle. The result is the following

$$\mathcal{L}^{(\text{scal})} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}M^2\phi^2 - \frac{1}{2m^2}\partial_\mu\phi\Box\partial^\mu\phi - J\phi. \quad (23)$$

We recall, as we have already commented, that if we multiply the above Lagrangian by -1 , we will obtain the Lagrangian describing the usual scalar model. Therefore, the Lagrangian found via the correspondence principle is a convenient mathematical tool built out with the only purpose of allowing the computation of the electromagnetic potential energy through the use of a fictitious scalar field.

The propagator related to the model we are discussing can in turn be written in momentum space as

$$O^{-1} = \frac{m^2}{k^4 - k^2m^2 + m^2M^2}, \quad (24)$$

which implies that this system is endowed with two massive poles, i.e.,

$$m_+^2 \equiv \frac{m^2}{2} \left[1 + \sqrt{1 - \frac{4M^2}{m^2}} \right], \quad (25)$$

$$m_-^2 \equiv \frac{m^2}{2} \left[1 - \sqrt{1 - \frac{4M^2}{m^2}} \right]. \quad (26)$$

To avoid tachyons in the model we assume that

$$0 \leq \frac{4M^2}{m^2} \leq 1. \quad (27)$$

Three interesting models arise from the constraint (27):

- A. $M = 0$: Lee-Wick electrodynamics.
- B. $4M^2 = m^2$: A model in which the propagator has a pole of order 2 at $k^2 = \frac{m^2}{2}$.
- C. $0 < \frac{4M^2}{m^2} < 1$: A system containing two modes with different non vanishing masses m_+ and m_- .

We discuss each one of them in the following.

A. $M = 0$

In this case the model describes the celebrated Lee-Wick electrodynamics [9, 10]. Since this system has been considered in detail in Ref. 1, the main results of the research are summarized below.

1. The potential energy for the interaction of two static pointlike electric charges Q_1 and Q_2 , is given for $D = 2, 4, 5, \dots$ by

$$E_D^{(\text{electr})}(r) = \frac{Q_1 Q_2}{(2\pi)^{\frac{D-1}{2}}} \left[\frac{2^{\frac{D-5}{2}} \Gamma(\frac{D-3}{2})}{r^{D-3}} - \left(\frac{m}{r}\right)^{\frac{D-3}{2}} K_{\frac{D-3}{2}}(mr) \right], \quad (28)$$

where K_ν is the modified Bessel function of the second kind of the order ν ; whereas for $D = 3$

$$E_3^{(\text{electr})}(r) = -\frac{Q_1 Q_2}{2\pi} \left[\ln \frac{r}{r_0} + K_0(mr) \right], \quad (29)$$

where Γ is the gamma function, and r_0 is an infrared regulator.

2. Both (28) and (29) agree asymptotically with the Coulomb potential energy at large distances.
3. For $D = 3$ and $D = 4$ the higher derivatives present in the model are able to tame the wild Coulombian divergence at the origin, while for $D = 2$, $E_2^{(\text{electr})}(r) = -Q_1 Q_2 \left[\frac{r}{2} + \frac{1}{2me^{mr}} \right]$ has a regular behavior at the origin. Unluckily, for $D > 4$ these higher derivatives are unable to control this singularity at $r = 0$.

B. $4M^2 = m^2$

Now, the propagator (24) reduces to

$$O^{-1} = \frac{m^2}{\left(k^2 - \frac{m^2}{2}\right)^2}, \quad (30)$$

and, consequently,

$$D(k) = \frac{m^2}{\left(k^2 - \frac{m^2}{2}\right)^2}, \quad (31)$$

which implies that

$$D(\mathbf{k}) = \frac{m^2}{\left(\mathbf{k}^2 + \frac{m^2}{2}\right)^2}. \quad (32)$$

Therefore,

$$E_D^{(\text{electr})}(r) = \frac{Q_1 Q_2 m^2}{(2\pi)^{D-1}} \int d^{D-1}\mathbf{k} \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{(\mathbf{k}^2 + \frac{m^2}{2})^2}. \quad (33)$$

Appealing to Appendix A, we promptly obtain

$$E_D^{(\text{electr})}(r) = \frac{Q_1 Q_2}{(2\pi)^{\frac{D-1}{2}}} \frac{m^2}{r^{\frac{D-3}{2}}} \int_0^\infty \frac{x^{\frac{D-1}{2}}}{(x^2 + \frac{m^2}{2})^2} J_{\frac{D-3}{2}}(xr) dx,$$

where J_ν is the Bessel function of the first kind.

On the other hand, taking into account that

$$\int_0^\infty \frac{J_\nu(bx) x^{\nu+1}}{(x^2 + a^2)^{\mu+1}} dx = \frac{a^{\nu-\mu} b^\mu}{2^\mu \Gamma(\mu+1)} K_{\nu-\mu}(ab), \quad (34)$$

where $-1 < \nu < (2\mu + \frac{3}{2})$, we arrive at the conclusion that

$$E_D^{(\text{electr})}(r) = \frac{Q_1 Q_2 m^{\frac{D-1}{2}}}{2^{\frac{3(D-1)}{4}} \pi^{\frac{D-1}{2}} r^{\frac{D-5}{2}}} K_{\frac{D-5}{2}}\left(\frac{mr}{\sqrt{2}}\right), \quad (35)$$

where $2 < D < 10$.

Keeping in mind that $K_\nu(r) \sim \sqrt{\frac{\pi}{2}} \frac{e^{-r}}{\sqrt{r}} \left(1 + \mathcal{O}\left(\frac{1}{r}\right)\right)$ for $r \rightarrow \infty$, it is straightforward to see that (35) and the Coulomb potential energy agree asymptotically.

Let us then study the behavior of the potential energy computed above (see (35)) at short distances ($r \rightarrow 0$). To accomplish this task, we must consider two different situations:

A. ν is equal to an integer plus one-half ($\nu = n + \frac{1}{2}$), which implies that

$$K_{n+\frac{1}{2}}(x) = \sqrt{\frac{\pi}{2x}} e^{-x} \sum_{k=0}^n \frac{(n+k)!}{k!(n-k)!(2x)^k}; \quad (36)$$

$$\text{accordingly, } K_{\pm\frac{1}{2}}(x) = \sqrt{\frac{\pi}{2x}} e^{-x}.$$

B. ν is an integer, in which case

$$\begin{aligned} K_\nu(x) = & (-1)^{\nu-1} \ln\left(\frac{x}{2}\right) \left(\frac{x}{2}\right)^\nu \sum_{k=0}^{\infty} \frac{(\frac{x}{2})^{2k}}{k!(k+\nu)!} + \\ & \frac{1}{2} \left(\frac{x}{2}\right)^\nu \sum_{k=0}^{\nu-1} \frac{(-1)^k (\nu-k-1)!}{k!} \left(\frac{x}{2}\right)^{2k} + \\ & \frac{(-1)^\nu}{2} \left(\frac{x}{2}\right)^\nu \sum_{k=0}^{\infty} \frac{\psi(k+1) + \psi(k+\nu+1)}{k!(k+\nu)!} \\ & \times \left(\frac{x}{2}\right)^{2k}, \end{aligned}$$

where $\psi(x) \equiv \frac{d}{dx} \ln \Gamma(x)$ is the psi function. As a consequence, if $\nu = 0$.

$$K_0(x) = \ln \frac{1}{x} + \dots$$

Here the dots denote terms remaining finite at $x = 0$.

Accordingly, if $D = 3, 4$ the potential is finite at the origin, whereas if $D = 5, 6, \dots, 9$ it has a singularity at $r = 0$.

In Fig. 1 it is shown the behavior of the potential energy for the $D = 3, 4$ — the cases where there are no singularities at the origin — while in Fig. 2 is depicted the potential energy for all the remaining values of D (5, 6, ..., 9). The reason for drawing two graphs was to emphasize the difference between the non singular and singular situations. Note, that in the singular cases, $E_D^{(\text{electr})}$ approaches the singularity more slowly as D increases.

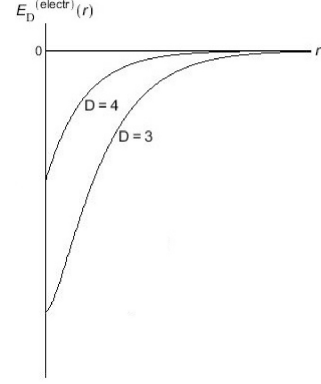


FIG. 1. Potential energy lacking singularity at the origin.

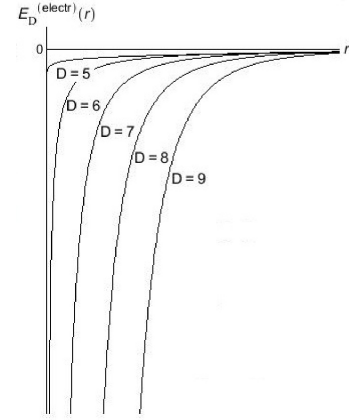


FIG. 2. Potential energy with singularity at the origin.

$$\text{C. } 0 < \frac{4M^2}{m^2} < 1$$

Here,

$$\begin{aligned} E_D^{(\text{electr})}(r) = & \frac{Q_1 Q_2}{(2\pi)^{D-1}} \frac{1}{\sqrt{1 - \frac{4M^2}{m^2}}} \int d^{D-1} \mathbf{k} e^{i\mathbf{k} \cdot \mathbf{r}} \left[\frac{1}{\mathbf{k}^2 + m_-^2} \right. \\ & \left. - \frac{1}{\mathbf{k}^2 + m_+^2} \right]. \end{aligned} \quad (37)$$

Following the same steps as above, we promptly obtain

$$E_D^{(\text{electr})}(r) = \frac{Q_1 Q_2}{(2\pi)^{\frac{D-1}{2}}} \frac{1}{\sqrt{1 - \frac{4M^2}{m^2}}} \left[\left(\frac{m_-}{r} \right)^{\frac{D-3}{2}} K_{\frac{D-3}{2}}(m_- r) - \left(\frac{m_+}{r} \right)^{\frac{D-3}{2}} K_{\frac{D-3}{2}}(m_+ r) \right], \quad 2 < D < 6. \quad (38)$$

It is straightforward to see that (38) coincides asymptotically with the Coulombian potential energy at great distances.

On the other hand, if $D = 3, 4$ the potential energy has no singularity at the origin, while if $D = 5$ it diverges at this point.

Fig. 3 shows the behavior of $E_D^{(\text{electr})}$ for $D = 3, 4, 5$.

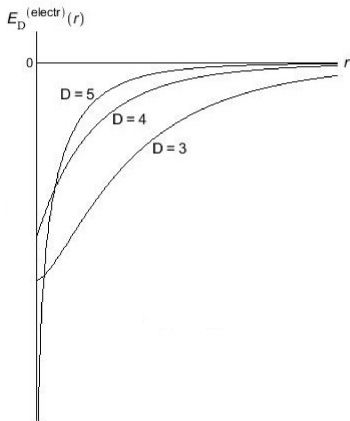


FIG. 3. Singular and non singular potential energy at the origin.

IV. GHOSTS IN HIGHER-DERIVATIVE THEORIES

The construction of regularized electrodynamics via the introduction of higher-order derivatives, was considered by Bopp [34], Landé [35–37], and Podolsky [38–41], a long time ago. Currently, this method is employed in the regularization of both gauge [42] and supersymmetric [43] theories; higher-order derivatives are also a common ingredient in string theory [44].

It is interesting to recall that to avoid divergences inherent in QED at short distances or, equivalently, at higher energies, we may introduce, for instance, a cutoff which renders the mass and charge of the electron finite. Indeed, consider in this spirit the Pauli-Villars regularization scheme used to obtain the electron self-energy. This prescription consists in cutoffing the integrals by assuming the existence of an auxiliary particle of heavy mass m . The propagator becomes modified by

$$-\frac{\eta_{\mu\nu}}{k^2} \rightarrow \frac{\eta_{\mu\nu}}{k^2} \frac{m^2}{k^2 - m^2} = -\frac{\eta_{\mu\nu}}{k^2} + \frac{\eta_{\mu\nu}}{k^2 - m^2}. \quad (39)$$

The mass of the particle is related to a cutoff l , which tames the infinities of the theory, by $l = \frac{1}{m}$. As the cutoff goes to zero, the mass of the auxiliary particle tends to infinity so that the unphysical fermion decouples from the system. The above result clearly shows that an electromagnetic theory having a propagator equal to the that given by (39) must have a better behavior at short distances than the usual QED. On the other hand, it is easy to show that after adding a gauge-fixing Lagrangian, $\mathcal{L}_{\text{gf}} = -\frac{1}{2\lambda}(\partial_\mu A^\mu)^2$, where λ is a gauge parameter, to the Lagrangian defining Lee-Wick theory (see the preceding section), i.e.,

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{4m^2}F_{\mu\nu}\square F^{\mu\nu}, \quad (40)$$

the resulting propagator,

$$D_{\mu\nu}(k) = \frac{m^2}{k^2(k^2 - m^2)} \left[\eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \left(1 + \lambda \left(\frac{k^2 - m^2}{m^2} \right) \right) \right],$$

coincides with (39) if the latter is sandwiched between conserved currents. Accordingly, the higher-order term of Lee-Wick Lagrangian modifies Maxwell Lagrangian only at short distances, improving its ultraviolet behavior.

Unfortunately, there is a widespread prejudice against higher-order theories. In fact, many physicists have a strong, although unreasonable, bias towards these models. In general, two arguments are invoked to discard these theories:

- A. Higher-order systems are always plagued by ghosts.
- B. Ostrogradski's no-go theorem.

We do our best in the following to make these subjects more clear; in addition, we discuss whether the four-dimensional version of the model found in Sec. III is infested by ghosts and analyze why the Coulomb singularity is lacking in this system.

A. Demystifying the wrong idea that all higher-order models are haunted by ghosts

Contrary to popular belief, not all higher-derivative systems are infected by ghosts. The following examples make clear that the idea that higher-order models are always haunted by ghosts is fallacious. To avoid being prolix, we restrict our discussion to gravitational and electromagnetic higher-derivative models.

1. Higher-derivative gravity models

In (2+1) dimensions, the BHT model (“new massive gravity”), which is defined by the Lagrangian

$$\mathcal{L} = \sqrt{g} \left[-\frac{2R}{\kappa^2} + \frac{2}{\kappa^2 m_2^2} \left(R_{\mu\nu}^2 - \frac{3}{8} R^2 \right) \right],$$

where $\kappa^2 = 4\kappa_3$, with κ_3 being Einstein’s constant in (2+1) dimensions, and $m_2(>0)$ is a mass parameter, has no ghosts at the tree level [45–48]. Interestingly enough, $R + R^2$ gravity in $(N+1)$ dimensions, i.e., the model defined by the Lagrangian $\mathcal{L} = \sqrt{(-1)^{N-2}g} \left[\frac{2R}{\kappa^2} + \frac{\alpha}{2} R^2 \right]$, where $\kappa^2 = 4\kappa_{N+1}$, with κ_{N+1} being Einstein’s constant in $(N+1)$ dimensions, and α is a free parameter, is also tree-level unitary [49].

It is worth noting, that there exist models containing ghosts that there are harmless. An interesting example is found in the models studied by Sotiriou and Faraoni: the so called $f(R)$ theories of gravity in $(3+1)$ dimensions. These authors analyzed these systems at the the classical level and came to the conclusion that “theories of the form $f(R, R^2, R_{\mu\nu}^2)$, contains, in general, a massive spin-2 ghost field in addition to the usual massless graviton and the massive scalar” [50]. Nevertheless, at the linear level, these theories are stable [2]. The reason why they do not explode is because the ghost cannot accelerate owing to energy conservation. Another way of seeing this is by analyzing the free wave solutions. Accordingly, the linear higher-derivative gravity model defined by linearizing the Lagrangian

$$\mathcal{L}_1 = \sqrt{-g} \left[\frac{2}{\kappa^2} R + \frac{\alpha}{2} R^2 + \frac{\beta}{2} R_{\mu\nu}^2 \right], \quad (41)$$

where $\kappa^2 = 4\kappa_4$, with κ_4 being Einstein’s constant in (3+1) dimensions, and α and β are free dimensionless coefficients, is not in disagreement with the result found by Sotiriou and Faraoni; indeed, despite containing a massive spin-2 ghost, as asserted by these researchers, the alluded ghost cannot cause trouble [4].

Recently it was shown that at least in the cases of specific cosmological backgrounds, the unphysical massive ghost present in the spectrum of higher-derivative gravity in $(3+1)$ dimensions is not growing up as a physical excitation and remains in the vacuum state until the initial frequency of the perturbation is close to the Planck scale. Consequently, higher-derivative models of quantum gravity can be seen as very satisfactory effective theories of quantum gravity below the Planck cut-off [3].

2. Higher-order electromagnetic systems

We begin by proving that Lee-Wick electrodynamics, although being infected by a ghost, is tree-level unitary

at familiar scales. To accomplish this task, we make use of a method pioneered by Veltman [51] that has been extensively used since it was conceived. The prescription consists in saturating the propagator with external currents and computing afterward the residues at all the poles of the alluded saturated propagator (SP). If the residues at the poles are positive or null, the model is tree-level unitary, but if at least one of the residues is negative, the system is nonunitary at the tree level.

The saturated propagator in momentum space is in turn given by

$$\begin{aligned} SP(k) &= J_\mu(k) D^{\mu\nu}(k) J_\nu(k) \\ &= -\frac{J^\mu(k) J_\mu(k)}{k^2} + \frac{J^\mu(k) J_\mu(k)}{k^2 - m^2}. \end{aligned} \quad (42)$$

Here the external current is conserved.

Let us then suppose that $k^2 \ll m^2$. Consequently,

$$SP(k) = J_\mu J^\mu \left[-\frac{1}{k^2} \right] + \mathcal{O}\left(\frac{k^2}{m^2}\right).$$

Now, bearing in mind that

$$\left(J^\mu J_\mu \right) \Big|_{k^2=0} < 0 \text{ (see Ref.[49])},$$

we come to the conclusion that

$$Res(SP)|_{k^2=0} > 0.$$

Therefore, at the scale at hand, Lee-Wick model is unitary at the tree level and, as a consequence, the massive spin-1 ghost is completely harmless.

We discuss now the tree-level unitarity of a higher-derivative spin-1 model in $(3+1)$ dimensions built out by Dalmazi and Santos [52]. The aforementioned system is of the Maxwell-Proca type and is defined by the Lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 [\partial H] + \frac{m^2}{2} (\partial^\nu H_{\mu\nu})^2 + H_{\mu\nu} J^{\mu\nu}, \quad (43)$$

where $H_{\mu\nu}$ is a symmetric rank-two tensor, $F_{\mu\nu}[\partial H] = \partial_\mu (\partial^\alpha H_{\nu\alpha}) - \partial_\nu (\partial^\alpha H_{\mu\alpha})$, and $J^{\mu\nu} = J^{\nu\mu}$ is the external current term which is not conserved. Note that if $J^{\mu\nu} = 0$, \mathcal{L} is invariant under any local transformation preserving $\partial^\alpha H_{\alpha\beta}$; as a result, \mathcal{L} is invariant under the gauge transformation $\delta_B H_{\mu\nu} = \partial^\sigma \partial^\rho B_{\mu\sigma\rho\nu}$, having the gauge parameters $B_{\mu\sigma\rho\nu}$ the same index symmetries of the Riemann tensor [53].

Making the source term equal to zero and adding the gauge-fixing term $\frac{\lambda}{2} G_{\mu\nu}^2$ to the resulting Lagrangian, where

$$G_{\mu\nu}(H) \equiv \square H_{\mu\nu} - 2\partial^\alpha \partial_{(\mu} H_{\nu)\alpha} + \eta_{\mu\nu} \partial^\alpha \partial^\beta H_{\alpha\beta},$$

leads to

$$\mathcal{L} = \frac{1}{2} H^{\mu\nu} O_{\mu\nu,\alpha\beta} H^{\alpha\beta}, \quad (44)$$

where

$$O(k) = \lambda k^4 P^{(2)} + \frac{k^2}{2}(-k^2 + m^2)P^{(1)} + \lambda k^4 P^{(0-s)} \\ + k^2((D-1)\lambda k^2 + m^2)P^{(0-w)} \\ + \sqrt{D-1}\lambda k^4(P^{(0-sw)} + P^{(0-ws)}),$$

wherein $\{P^{(2)}, P^{(1)}, P^{(0-s)}, P^{(0-w)}, P^{(0-sw)}, P^{(0-ws)}\}$ is the set of the usual D -dimensional Barnes-Rivers operators [54].

Accordingly, the propagator in momentum space reads

$$O^{-1}(k) = i \left[\frac{1}{\lambda k^4} P^{(2)} - \frac{2}{k^2(k^2 - m^2)} P^{(1)} + \frac{1}{k^2 m^2} P^{(0-w)} \right. \\ \left. + \left(\frac{1}{\lambda k^4} + \frac{D-1}{k^2 m^2} \right) P^{(0-s)} - \frac{\sqrt{D-1}}{k^2 m^2} \left(P^{(0-sw)} \right. \right. \\ \left. \left. + P^{(0-ws)} \right) \right]. \quad (45)$$

Now, since the external current is not conserved, neither $k_\mu J^{\mu\nu}$ nor $k_\nu J^{\mu\nu}$ are null. Nonetheless, since the gauge symmetry is such that $\delta_B \partial^\nu H_{\mu\nu} = 0$, the invariance of the source term $\delta_B \int d^D x H_{\mu\nu} J^{\mu\nu} = 0$ requires $J_{\mu\nu} = \partial_\mu J_\nu - \partial_\nu J_\mu$, which in momentum space assumes the form $J_{\mu\nu}(k) = i(k_\mu J_\nu - k_\nu J_\mu)$. It follows then the saturated propagator can be written as

$$SP(k) = J_{\mu\nu}^* (O^{-1})^{\mu\nu, \kappa\lambda} J_{\kappa\lambda} \\ = i J_{\mu\nu}^* \left[\frac{-2P^{(1)}}{k^2(k^2 - m^2)} + \frac{P^{(0-w)}}{m^2 k^2} \right]^{\mu\nu, \kappa\lambda} J_{\kappa\lambda} \\ = 4i \left(- \frac{J_\mu^* \theta^{\mu\nu} J_\nu}{k^2 - m^2} + \frac{J_\mu^* w^{\mu\nu} J_\nu}{m^2} \right). \quad (46)$$

The last line of (45) corresponds exactly to the two-point amplitude for the Maxwell-Proca model with non conserved external currents. Note that the pole at $k^2 = 0$ cancels out and the imaginary part of the residue at $k^2 = m^2$ is of course positive [55], which guarantees the tree-level unitarity of this higher-derivative Maxwell-Proca theory.

We remark that owing to the fact that the external current in momentum space related to the example above involves the presence of the imaginary unit i , the Veltman prescription used in the first example had to be reformulated as follows:

1. Add an i to the propagator.
2. Construct the saturated propagator according to the following recipe:

$$SP = (\text{external current})^* (\text{propagator}) (\text{external current}).$$

3. Compute the residues at all the poles of the imaginary part of SP ; if these residues are positive or null, the model is tree-level unitary, but if at least one of the residues is negative, the system is non unitary at the tree level.

B. Demystifying Ostrogradski's no-go theorem

Let us then discuss the common misconception that *singular* higher-derivative models can be discarded by appealing to Ostrogradski's no-go theorem. For the sake of generality consider a higher-derivative system in D dimensions. According to popular belief, Ostrogradski's result implies that there exists a linear instability in the Hamiltonian associated with all higher-derivative systems. This is a completely untrue assertion. Indeed, Ostrogradski only treated non singular models [56]. Therefore, the only way of circumventing Ostrogradski's no-go theorem is by considering singular models, which is in accord with the conclusion reached by Woodard [57]. An interesting example of this kind is the rigid relativistic particle studied by Plyushchay [58].

As is well known, all higher-derivative gauge models are singular; as a result they can not be discarded *a priori* by Ostrogradski's theorem since it does not apply to them. This does not mean, of course, that they are always ghost-free systems. In subsections 1 and 2, we exhibited some interesting higher-derivative gauge models that are tree-level unitary and obviously do not violate Ostrogradski's theorem.

C. Counting ghosts

We investigate now whether the version in four dimensions of the model discussed in Sec. 3 is plagued by ghosts. This model, as it was shown in the aforementioned section, comprises three different cases which were obtained by imposing that this system has no tachyons. We investigate these situations below.

1. $M = 0$

Since in this case we are contemplating Lee-Wick electrodynamics which is gauge invariant, Ostrogradski's theorem cannot be used to throw away this system. However, as we have already commented, this does not mean that it is lacking ghosts. A quick glance at (41) is enough to allow us to write

$$Res(SP)|_{k^2=0} > 0, \quad Res(SP)|_{k^2=m^2} < 0. \quad (47)$$

Therefore, Lee-Wick electrodynamics is non unitary at the tree level; but if $k^2 \ll m^2$, this electrodynamics is unitary at this scale (see Sec. IV). The latter situation is an example of a model that despite having a ghost is

tree-level unitary. Actually, this ghost does not cause any trouble.

We consider now the problem of the Coulombian singularity in Lee-Wick electrodynamics. Interestingly enough, it seems that there is a simple relation between the renormalizability of Lee-Wick theory and the absence of the Coulombian singularity at the classical level: the renormalizability implies that the potential energy is singularity free at the origin. This is in accord with a conjecture that appeared recently in the literature that says that renormalizable higher-order models have a classical potential energy lacking singularities at the origin and are nonunitary [59].

Indeed, from (28) we find that the modified Coulombian energy is

$$E_4^{(\text{electr})}(r) = \frac{Q_1 Q_2}{4\pi} \left[\frac{1}{r} - \frac{e^{-mr}}{r} \right]. \quad (48)$$

Expanding the exponential at the origin $r = 0$ into powers series, it is easy to check that the contribution of the higher-derivative term to the Coulombian energy make it regular. The modified potential energy tends to the constant value

$$E_4^{(\text{electr})}(r) = \frac{Q_1 Q_2}{4\pi} \left[m + \mathcal{O}(r) \right]. \quad (49)$$

The singularity cancellation occurs because the zero order term of the Yukawa energy produces the coefficient -1 that cancels out the original Coulomb term.

Therefore, bearing in mind that the Lee-Wick model is renormalizable, it agrees, of course, with the cited conjecture.

$$2. \quad 4M^2 = m^2$$

The propagator concerning this system reads

$$\begin{aligned} O_{\mu\nu}^{-1}(k) &= i \left[\frac{m^2}{(k^2 - \frac{m^2}{2})^2} \theta_{\mu\nu} + \frac{4}{m^2} \omega_{\mu\nu} \right] \\ &= i \left[\frac{m^2}{(k_0^2 - \omega^2)^2} \theta_{\mu\nu} + \frac{4}{m^2} \omega_{\mu\nu} \right], \end{aligned} \quad (50)$$

where $\omega^2 \equiv \mathbf{k}^2 + \frac{m^2}{2}$.

Accordingly, the saturated propagator can be written as

$$\begin{aligned} SP(k_0, \mathbf{k}) &= i J_\mu^*(k_0, \mathbf{k}) \frac{m^2}{(k_0^2 - \omega^2)^2} J^\mu(k_0, \mathbf{k}) \\ &= i J_\mu^*(k_0, \mathbf{k}) \frac{m^2}{(k_0 - \omega)^2 (k_0 + \omega)^2} J^\mu(k_0, \mathbf{k}), \end{aligned}$$

which implies that $SP(k_0, \mathbf{k})$ has two poles of order two: one at $k_0 = \omega$ and the other at $k_0 = -\omega$. As a result,

$$Res(SP)|_{k_0=\omega} = \left[\frac{d}{dk_0} \left(\frac{i J_\mu^* J^\mu}{(k_0 + \omega)^2} \right) \right]_{k_0=\omega}, \quad (51)$$

$$Res(SP)|_{k_0=-\omega} = \left[\frac{d}{dk_0} \left(\frac{i J_\mu^* J^\mu}{(k_0 - \omega)^2} \right) \right]_{k_0=-\omega}. \quad (52)$$

In order to evaluate (51) and (52), we need beforehand the expression of the specific physical current concerning the model to be analyzed. Nevertheless, a way to get around this difficulty is to appeal to the conjecture mentioned in the preceding subsection. In fact, since this system is renormalizable, it is non unitary as far as this conjecture is concerned, which also tells us that the classical potential energy is finite at the origin. Let us prove this late assertion. From (35), we find

$$E_4^{(\text{electr})}(r) = \frac{Q_1 Q_2 m^{\frac{3}{2}}}{2^{\frac{3}{2}} \pi^{\frac{3}{2}} r^{-\frac{1}{2}}} K_{\frac{1}{2}} \left(\frac{mr}{\sqrt{2}} \right). \quad (53)$$

Therefore, for $r \rightarrow 0$ the preceding result assumes the form

$$E_4^{(\text{electr})}(r) = \frac{Q_1 Q_2 m}{4\sqrt{2}\pi} + \mathcal{O}(r), \quad (54)$$

which, of course, has no singularity at the origin.

$$3. \quad 0 < \frac{4M^2}{m^2} < 1$$

Bearing in mind that the propagator for this system is given by

$$(O^{-1})_{\mu\nu} = \left[\frac{1}{k^2 - m_+^2} - \frac{1}{k^2 - m_-^2} \right] \frac{\theta_{\mu\nu}}{\sqrt{1 - \frac{4M^2}{m^2}}} + \frac{\omega_{\mu\nu}}{m^2},$$

we immediately find that

$$Res(SP)|_{k^2=m_+^2} < 0, \quad Res(SP)|_{k^2=m_-^2} > 0. \quad (55)$$

Thus, the model is nonunitary; in addition, the potential energy is finite at the origin and equal to $E_4^{(\text{electr})}(0) = \frac{Q_1 Q_2}{4\pi \sqrt{1 - \frac{4M^2}{m^2}}} (m_+ - m_-)$.

4. Comment

We have found that the four-dimensional electromagnetic models described above are renormalizable, nonunitary and have a non-singular classical potential energy at the origin, which agrees with a conjecture recently formulated [59]. We remark that Stelle [60] was the first to hint at the possibility of existing a simple relation between the renormalizability of a higher-derivative quantum theory and the absence of a singularity at the origin concerning the classical potential energy. Recently this subject was also discussed in [61, 62]. Note, however that in [59], it is surmised that if a higher-order quantum theory is renormalizable, it has a classical potential energy finite at the origin and, besides, it is also nonunitary.

V. FINAL REMARKS

We have developed a simple prescription for computing the interaction potential energy between two point like static charges in the framework of D -dimensional electromagnetic models from the corresponding scalar ones, via a correspondence principle that connects the electromagnetic and scalar fields. The key point of the method consists in finding the “scalar propagator” $D(\mathbf{k})$, which is a quite trivial computation. The interparticle potential energy can then be easily calculated through the expression

$$E_D^{(\text{elctr})} = \frac{Q_1 Q_2}{(2\pi)^{D-1}} \int d^{D-1} \mathbf{k} e^{i\mathbf{k} \cdot \mathbf{r}} D(\mathbf{k}). \quad (56)$$

This prescription allows also, as an added bonus, to compute the D -dimensional potential energy for the interaction of two scalar charges through the formula

$$E_D^{(\text{standscal})}(r) = -\frac{\sigma_1 \sigma_2}{(2\pi)^{D-1}} \int d^{D-1} \mathbf{k} e^{i\mathbf{k} \cdot \mathbf{r}} D(\mathbf{k}). \quad (57)$$

The method was used afterward to obtain the D -dimensional potential energy regarding a higher-derivative electromagnetic model, being its behavior discussed at large as well small distances. It was found that the four-dimensional systems that comprised the mentioned model and which were obtained by requiring that they were non tachyonic, are renormalizable, nonunitary, and have a potential energy that has no singularity at the origin. These results, as we have already mentioned, are in complete accord with a conjecture that recently appeared in the literature [59].

It is interesting to note that if we have used the first order formalism to compute the potential energy at the origin alluded above, we would have come to the conclusion that the mentioned result is singularity free as well. Therefore, both prescriptions lead to the same conclusion. Indeed, consider, for instance, Lee-Wick electrodynamics (see (20)). The field theory with real vectorial fields A_μ and Z_μ with Lagrangian

$$\mathcal{L} = \frac{1}{2} A_\mu \square Z^\mu + \frac{1}{2} \partial_\mu A^\mu \partial_\nu Z^\nu - \frac{m^2}{8} A_\mu A^\mu + \frac{m^2}{4} A_\mu Z^\mu - \frac{m^2}{8} Z_\mu Z^\mu, \quad (58)$$

is equivalent to the field theory with the Lagrangian (20). In fact, varying Z_μ gives

$$Z_\mu = A_\mu + \frac{2}{m^2} \square A_\mu - \frac{2}{m^2} \partial_\mu \partial_\nu A^\nu, \quad (59)$$

and the coupled second-order equations from (58) are fully equivalent to the fourth-order equations from (20). The system (58) now separates clearly into the Lagrangians for two fields, when we make the change of variables $A_\mu = B_\mu + C_\mu$ and $Z_\mu = B_\mu - C_\mu$. In terms of B_μ , C_μ , $B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu$, and $C_{\mu\nu} \equiv \partial_\mu C_\nu - \partial_\nu C_\mu$, the Lagrangian now becomes,

$$\mathcal{L} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \frac{1}{4} C_{\mu\nu} C^{\mu\nu} - \frac{m^2}{2} C_\mu C^\mu, \quad (60)$$

which is nothing but the difference of the Maxwell Lagrangian for B_μ and the Proca Lagrangian for C_μ . Accordingly, the potential energy for the interaction of two static charges Q_1 and Q_2 computed using (60) coincides with (48), as expected, being as a consequence singularity free at the origin. We remark that Plyushchay has employed the first order formalism for dealing with mechanical systems with third derivatives [58, 63, 64].

Could the prescription we have devised be applied for the higher-derivative extension of the topologically massive electrodynamics build out by Deser and Jackiw [65]? The answer is affirmative provided the external current is conserved. It is remarkable that theories with higher derivative Chern-Simons extensions can be related to non-commutative geometry [66].

We then examined the important subject of ghosts in higher-derivative models in detail to clarify many prejudices against these systems. In particular, we demystified the wrong idea that all high order models are infested by ghosts as well as the no-go theorem by Ostrogradski.

Last but not least, we would like to compare the potential energies obtained in subsection C (Counting ghosts) of section IV with the Coulombian energy. To do that we drew the graphs related to these potential energies together with the Coulombian potential. These pictures clearly shows that except in case A (Lee-Wick electrodynamics), there is a great difference between the remaining potentials and the Coulomb one (see Fig 4). Why is this so? The answer, in a sense, is simple: Lee-Wick electrodynamics is the only linear forth-order gauge-invariant generalization of Maxwell electrodynamics [26]. As we have already shown, the higher-order term of Lee-Wick Lagrangian modifies Maxwell Lagrangian only at short distances, improving its ultraviolet behavior.

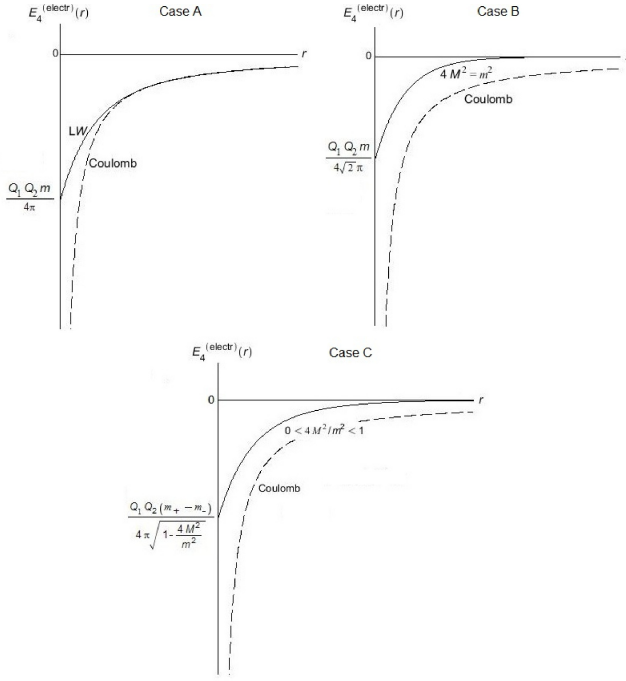


FIG. 4. A comparison between the potential energy and the Coulomb one for the systems found in subsection C (Counting ghosts) of section IV.

ACKNOWLEDGMENTS

The authors acknowledge financial support from CNPq and FAPERJ.

Appendix A: SOLVING INTEGRALS OF THE TYPE $\int d^{D-1}\mathbf{k} e^{i\mathbf{k}\cdot\mathbf{r}} f(|\mathbf{k}|)$

In order to find the D -dimensional interparticle potential energy related to the models dealt with in this paper, we have to compute $E_D^{(\text{electr})}(r)$ which can be generically written as

$$E_D^{(\text{electr})}(r) = \frac{Q_1 Q_2}{(2\pi)^{D-1}} \int d^{D-1}\mathbf{k} e^{i\mathbf{k}\cdot\mathbf{r}} f(|\mathbf{k}|). \quad (\text{A1})$$

In this Appendix we show how to convert this expression — which contains a $(D-1)$ -dimensional integral — into a formula including only a uni-dimensional integral, which greatly facilitates the computational stage.

To begin with we introduce the variable $\mathbf{x} \equiv \mathbf{k}$ and represent the symbol $|\mathbf{k}|$ by x . Using then the geometry

depicted in Fig. 5 and the infinitesimal volume element in spherical coordinates $(x, \theta_1, \dots, \theta_{D-2})$, i.e.,

$$d^{D-1}\mathbf{k} \equiv d^{D-1}\mathbf{x} = x^{D-2} dx \prod_{i=1}^{D-2} \sin^{D-2-i} \theta_i d\theta_i,$$

(A1) assumes the form

$$E_D^{(\text{electr})}(r) = \frac{Q_1 Q_2}{(2\pi)^{D-1}} \left[\int_0^\infty dx x^{D-2} f(x) \int_0^\pi d\theta_1 e^{ixr \cos \theta_1} \times \sin^{D-3} \theta_1 \right] F,$$

where

$$F \equiv \int_0^\pi d\theta_2 \sin^{D-4} \theta_2 \int_0^\pi d\theta_3 \sin^{D-5} \theta_3 \dots \int_0^\pi d\theta_{D-3} \sin \theta_{D-3} \int_0^{2\pi} d\theta_{D-2} = \frac{2\pi^{\frac{D-2}{2}}}{\Gamma\left(\frac{D-2}{2}\right)}.$$

Now, bearing in mind that

$$\int_0^\pi e^{i\beta \cos x} \sin^{2\nu} x dx = \sqrt{\pi} \left(\frac{2}{\beta}\right)^\nu \Gamma\left(\nu + \frac{1}{2}\right) J_\nu(\beta), \quad \nu > -\frac{1}{2},$$

we come to the conclusion that

$$E_D^{(\text{electr})}(r) = \frac{Q_1 Q_2}{(2\pi)^{\frac{D-1}{2}} r^{\frac{D-3}{2}}} \int_0^\infty x^{\frac{D-1}{2}} f(x) J_{\frac{D-3}{2}}(xr) dx,$$

where $D > 2$.

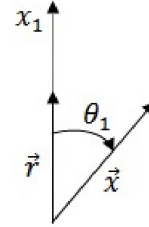


FIG. 5. Geometry for the computation of the integral (A1).

[1] A. Accioly, J. Helayël-Neto, F. E. Barone, F. A. Barone, and P. Gaete, Phys. Rev. D **90**, 105029 (2014).

[2] I. L. Shapiro, A. Pelinson and F. Salles, Mod. Phys. Lett. A **29**, 1430034 (2014).

- [3] F. Salles and I. L. Shapiro, Phys. Rev. D **89**, 084054 (2014).
- [4] A. Accioly, J. Helayël-Neto, B. Giacchini, and Wallace Herdy, Phys. Rev. D **91**, 125009 (2015).
- [5] M. Ostrogradski, Mem. Ac. St. Petersburg **6**, 385 (1850).
- [6] A. Zee, *Quantum Field Theory in a Nutshell*, 2nd ed. (Princeton University Press, Princeton, NJ) (2010).
- [7] L. Ryder, *Quantum Field Theory*, 2nd ed. (Cambridge University Press, Cambridge, England) (1996).
- [8] L. Schulman, *Techniques and Applications of Path Integration*, (Dover Publications, Inc. Mineola, N.Y.) (2005).
- [9] T. Lee and G. Wick, Nucl. Phys. B **9**, 209 (1969).
- [10] T. Lee and G. Wick, Phys. Rev. D **2**, 1033 (1970).
- [11] F. A. Barone, G. Flores-Hidalgo, and A. A. Nogueira, Phys. Rev. D **88**, 105031 (2013).
- [12] R. Cuzinato, C. de Melo, L. Medeiros, and P. Pompeia, Int. J. Mod. Phys. A **26**, 3641 (2011).
- [13] A. Accioly and E. Scatena, Mod. Phys. Lett. A **25**, 269 (2010).
- [14] A. Accioly, P. Gaete, J. Helayël-Neto, E. Scatena, and R. Turcati, Mod. Phys. Lett. A **26**, 1985 (2011).
- [15] A. Accioly, P. Gaete, J. Helayël-Neto, E. Scatena, and R. Turcati, arXiv:1012.1045v2.
- [16] A. Accioly, J. Helayël-Neto, B. Pereira-Dias, and C. Hernaski, Phys. Rev. D **86**, 105046 (2012).
- [17] C. Galvão and B. Pimentel, Can. J. Phys. **66**, 460 (1988).
- [18] B. Bufalo, B. Pimentel, and G. Zambrano, Phys. Rev. D **83**, 045007 (2011).
- [19] M. Bertin, B. Pimentel, and G. Zambrano, J. Math. Phys. (N.Y.) **52**, 102902 (2011).
- [20] J. Frankel, Phys. Rev. E **54**, 5859 (1996).
- [21] A. Zayats, arXiv:1306.3966.
- [22] R. B. B. Santos, Mod. Phys. Lett. A **26**, 1909 (2011).
- [23] S. Moyaedi, M. Setare, and B. Khosropour, Adv. High Energy Phys. **2013**, 657870 (2013).
- [24] S. Moyaedi, M. Setare, and B. Khosropour, Int. J. Mod. Phys. A **28**, 1350142 (2013).
- [25] P. Gaete, J. Helayël-Neto, and E. Spallucci, J. Phys. A textbf45, 215401 (2012).
- [26] R. Cuzinato, C. Melo, and P. Pompeia, Ann. Phys. **322**, 1211 (2007).
- [27] S. Kruglov, J. Phys. A **43**, 245403 (2010).
- [28] I. Cho and O. Kwon, Phys. Rev. D **82**, 025013 (2010).
- [29] A. Accioly and M. Dias, Phys. Rev. D **70**, 107705 (2004).
- [30] P. Gaete, Int. J. Phys. A **27**, 1250061 (2012).
- [31] P. Gaete and E. Spallucci, J. Phys. A **45**, 065401 (2012).
- [32] F. A. Barone and A. A. Nogueira, Eur. Phys. J. C **75**, 339 (2015).
- [33] F. A. Barone, G. Flores-Hidalgo, A. A. Nogueira, Phys. Rev. D **91**, 027701 (2015).
- [34] F. Bopp, Ann. Phys. **38**, 345 (1940).
- [35] A. Landé, Phys. Rev. **60**, 121 (1941).
- [36] A. Landé and L. Thomas, Phys. Rev. **60**, 514 (1941).
- [37] A. Landé and L. Thomas, Phys. Rev. **65**, 175 (1944).
- [38] B. Podolsky, Phys. Rev. **62**, 68 (1942).
- [39] B. Podolsky and C. Kikuchi, Phys. Rev. **65**, 228 (1944).
- [40] B. Podolsky and C. Kikuchi, Phys. Rev. **67**, 184 (1945).
- [41] B. Podolsky and P. Schwed, Rev. Mod. Phys. **20**, 40 (1948).
- [42] L. Faddeev and A. Slavnov, *Gauge Fields: Introduction to Quantum Theory*, 2nd ed (Frontiers in Physics Series, vol. 83, 1990), and references therein.
- [43] P. West, Nucl. Phys. B **268**, 125 (1986).
- [44] A. Polyakov, Nucl. Phys. B **268**, 406 (1986).
- [45] E. Bergshoeff, O. Holm, and P. Townsend, Phys. Rev. Lett. **102**, 201301 (2009).
- [46] S. Deser, Phys. Rev. Lett. **103**, 101302 (2009).
- [47] A. Accioly, J. Helayël-Neto, J. Morais, E. Scatena, and R. Turcati, Phys. Rev. D **83**, 104005 (2011).
- [48] A. Accioly, J. Helayël-Neto, E. Scatena, J. Morais, R. Turcati and B. Pereira-Dias Class. Quantum Grav. **28**, 225008 (2011).
- [49] A. Accioly, A. Azeredo, and H. Mukai, J. Math. Phys. **43**, 473 (2002).
- [50] T. Soritiou and V. Faraoni, Rev. Mod. Phys. **82**, 451 (2010).
- [51] M. Veltman, in *Methods in Field Theory*, eds. R. Balian and J. Zinn-Justin (North Holland/World Scientific, 1981).
- [52] D. Dalmazi and R. Santos, Phys. Rev. D **87**, 085021 (2013).
- [53] S. Deser and P. K. Townsend, Phys. Lett. B **98**, 188 (1981).
- [54] A. Accioly, J. Helayël-Neto, F. E. Barone, W. Herdy, Class. Quantum Gravity **32**, 035021 (2015).
- [55] D. Dalmazi and R. Santos, Phys. Rev. D **84**, 045027 (2011).
- [56] T. Nakamura and S. Hamamoto, Prog. Theor. Phys. **95**, 469 (1996).
- [57] R. Woodard, Lect. Notes Phys. **720**, 403 (2007).
- [58] M. Plyushchay, Mod. Phys. Lett. A **4**, 837 (1989).
- [59] A. Accioly, J. Helayël-Neto, E. Scatena, R. Turcati, Int. J. Mod. Phys. D **22**, 1342015 (2013) (honorable mention in the 2013 essay competition of the Gravity Research Foundation).
- [60] K. S. Stelle, Phys. Rev. D **16**, 953 (1977).
- [61] L. Modesto, T. de Paula Netto, I. L. Shapiro, JHEP **04** (2015) 098.
- [62] I. L. Shapiro, Phys. Lett. B **744**, 67 (2015).
- [63] M. Plyushchay, Phys. Lett. B **243**, 383 (1990).
- [64] M. Plyushchay, Nucl. Phys. B **362**, 54 (1991).
- [65] S. Deser and R. Jackiw, Phys. Lett. B **451**, 73 (1999).
- [66] M. Plyushchay, arXiv:math-ph/0604022v4 20 Apr 2006.